Numerical Propagation of VHE Cosmic Rays in the Galaxy

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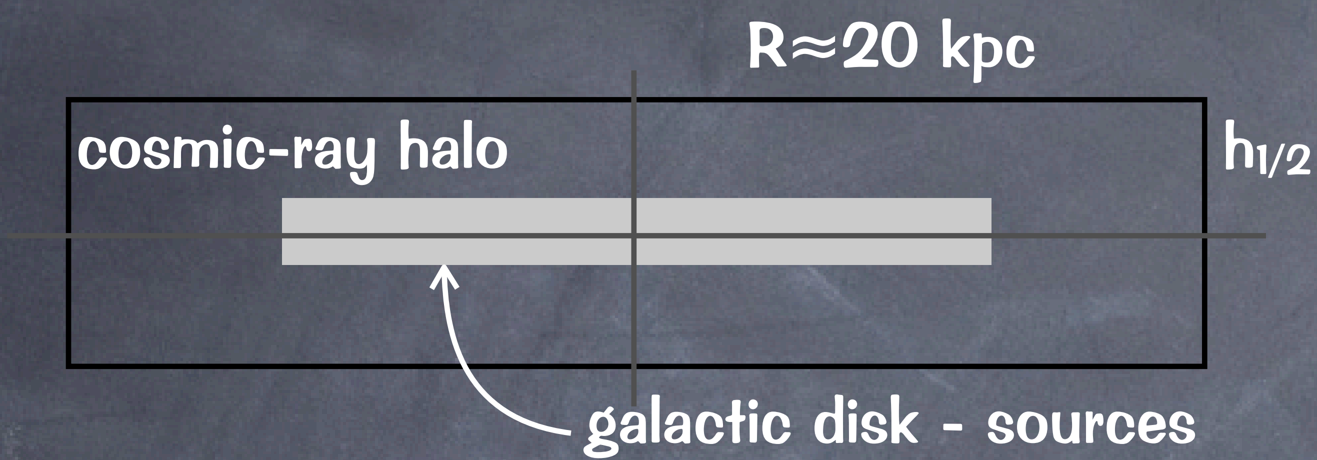
Outline

- "standard" model
- simulations
- diffusion and drifts
- toy model
- "realistic" models of the GMF

"Standard" Model

Ginzburg&Ptuskin 1976, Berezinskii at el. 1990...



CR density ~ $E^{-2.7}$

source sp. ~ $E^{-2\,..\,-2.4}$

$$\frac{N}{T} = Q$$

escape time ~ $E^{-0.3\,..\,-0.6}$

grammage $X = \rho\, v\, T$

1. source spectrum
2. production of light elements by spallation
3. anisotropies vs energy

diffusion model: $X \propto D^{-1}$

$D \sim 3\text{-}5\ 10^{28}\ cm^2/s$ @ GeV/n

- plain diffusion: $D \propto R^{0.6}$
- diffusion+reacceleration: $D \propto R^{0.3}$

e.g. GALPROP (Strong&Moskalenko 1998)

$X \approx 10\ g/cm^2$ @ GeV/n

$X \propto R^{-0.6}$ ← rigidity

open issues

- spectral exponent
- anisotropies

extrapolation issues

anisotropy
Hillas 2005

	1.5×10^{14}eV	10^{15}eV	1.5×10^{17}eV
obs.	0.037%	<0.4%	1.7%
$D\propto R^{0.6}$	5%	16%	180%
$D\propto R^{1/3}$	0.6%	1.1%	3.7%

residence time

observations: T(GeV) ~ 10^7 yr.

extrapolations: T(10^{16}eV) ~ 600 yr with $D\propto R^{0.6}$
~ 5×10^4 yr with $D\propto R^{1/3}$

simulations: Zirakashvili et al. 1998: 10^5 yr at 10^{17}eV
Horandel et al. 2007: 10^7 yr at 10^{15}eV
the slope is -1

Why go numerical*?

- around 10^{17}eV: transition region for protons
- simulations in literature obtain too longer times, and the slope seems odd too
- we would like to see the transition from -1/3 to -1
- "realistic" model of the galactic magnetic field: arms, gradients...
- non-constant background field: what happens to diffusion?

* simulation of trajectories

Numerical Simulation

- arbitrary magnetic field (regular + turbulent)
  - regular: constant, azimuthal, galactic...
  - turbulent: isotropic and slab turbulence
- diffusion, drifts automatically included
- can calculate diffusion coefficients, times of escape, anisotropies....
- minimum energy $\sim 10^{15}$eV for protons

Turbulent Field

1) FFT

Casse et al. 2001

normalization

amplitude
$|A(\boldsymbol{k})|^2 \propto k^{-\gamma-2}$

$$\delta B = \alpha \sum_{\boldsymbol{k}} \hat{\boldsymbol{\epsilon}}(\boldsymbol{k}) A(\boldsymbol{k}) \exp \frac{\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}}{L_{\max}}$$

wave-vectors
integer coordinates

versor $\perp$ k
$\nabla \cdot \delta B = 0$

memory $\sim$ N^3 $\Rightarrow$ limited dynamic range

time $\sim$ 1 $\Rightarrow$ faster

problems when $r_L \ll L_{min}$ & $r_L \gg L_{max}$

2) Plane Waves

Giacalone & Jokipii 1999

direction of n-th wave

$$[x', y', z'] = \mathcal{R}(\theta_n, \phi_n) \times [x, y, z]$$

$$\delta B = \sum_{n=1}^{N_m} A_{\boldsymbol{k}_n} \hat{\boldsymbol{\epsilon}}_n \exp(\mathrm{i}k_n z'_n + \mathrm{i}\beta_n)$$

$$\hat{\epsilon}_n = \cos\alpha_n \hat{\boldsymbol{x}}'_n + \mathrm{i}\sin\alpha_n \hat{\boldsymbol{y}}'_n$$

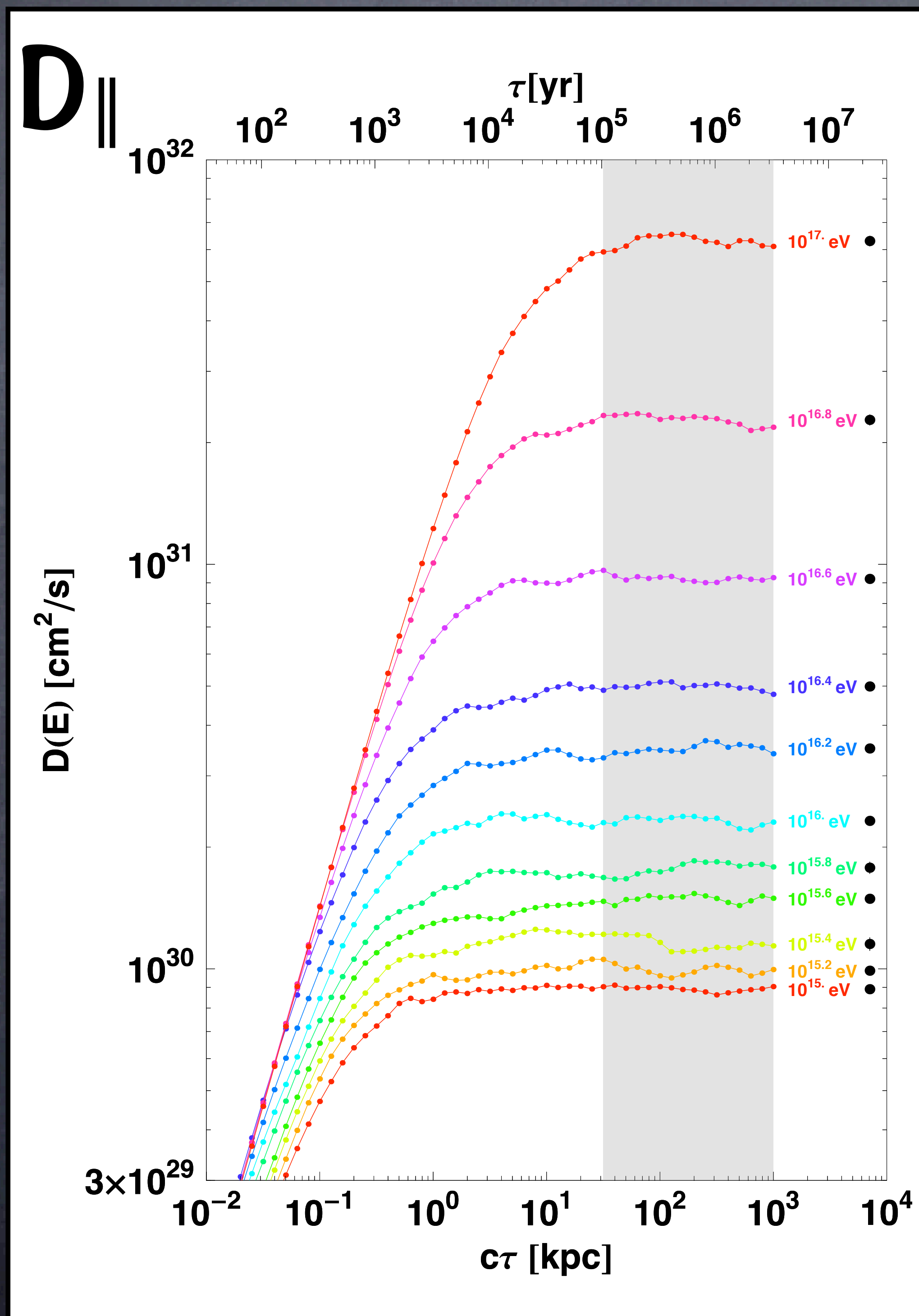
memory $\sim$ N_m $\Rightarrow$ "unlimited" dynamic range

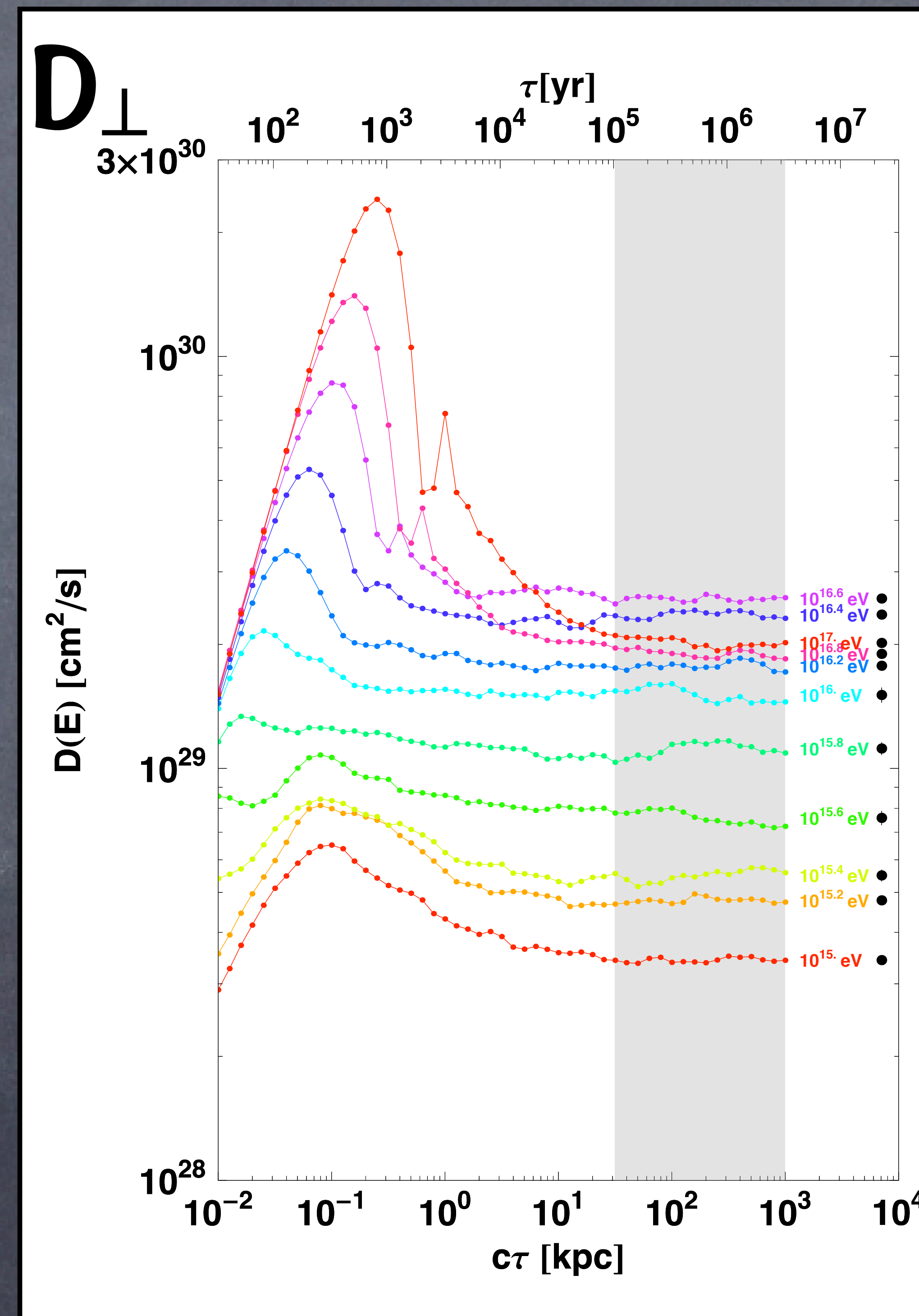
time $\sim$ N_m $\Rightarrow$ slower

N_m $\sim$ 100/decade (Parizot 2004)

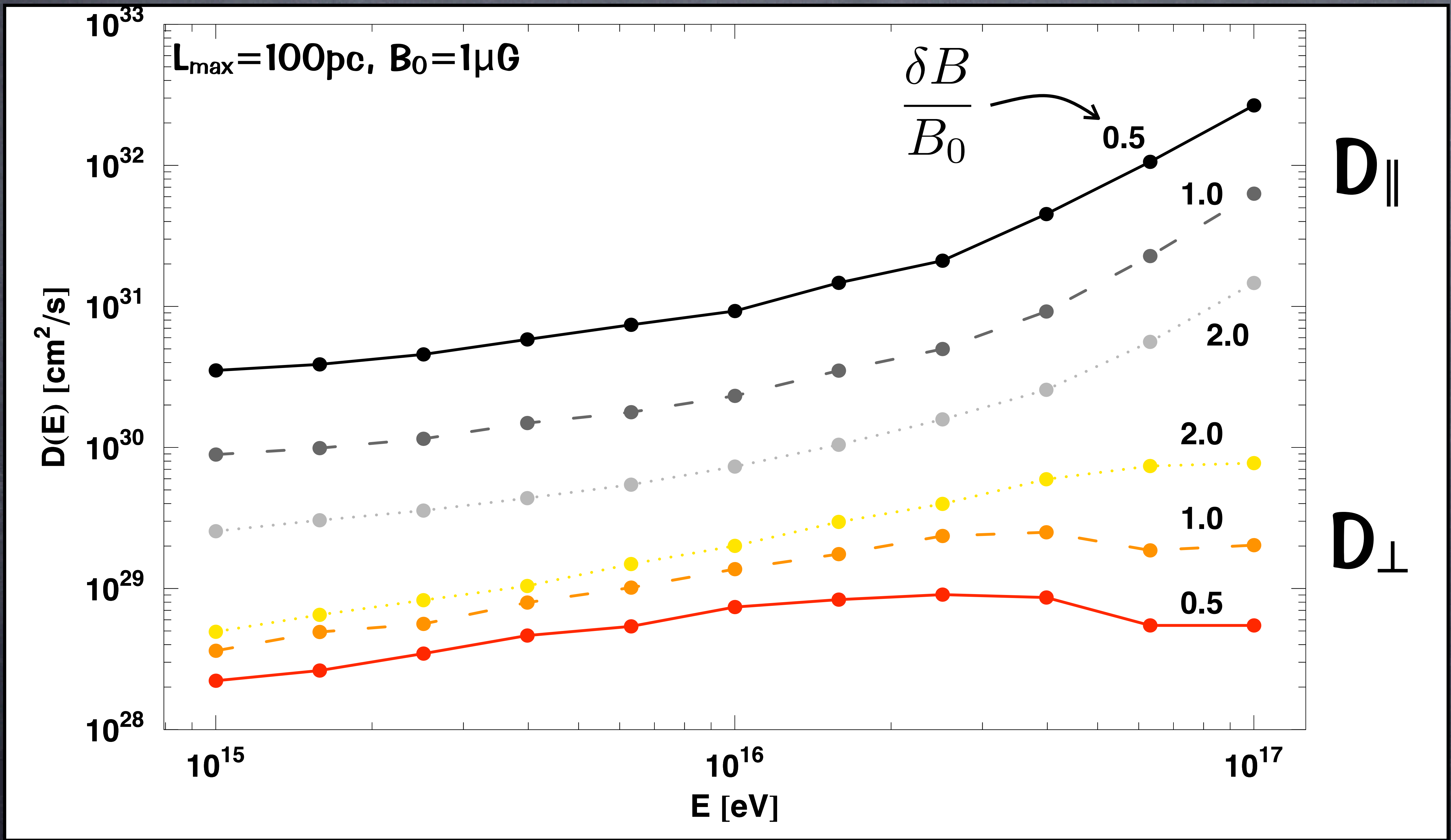
Diffusion Coefficients

$$\frac{\delta B}{B_0} = 1 \qquad D(\tau) = \frac{\langle \Delta^2(\tau) \rangle}{2\tau}$$

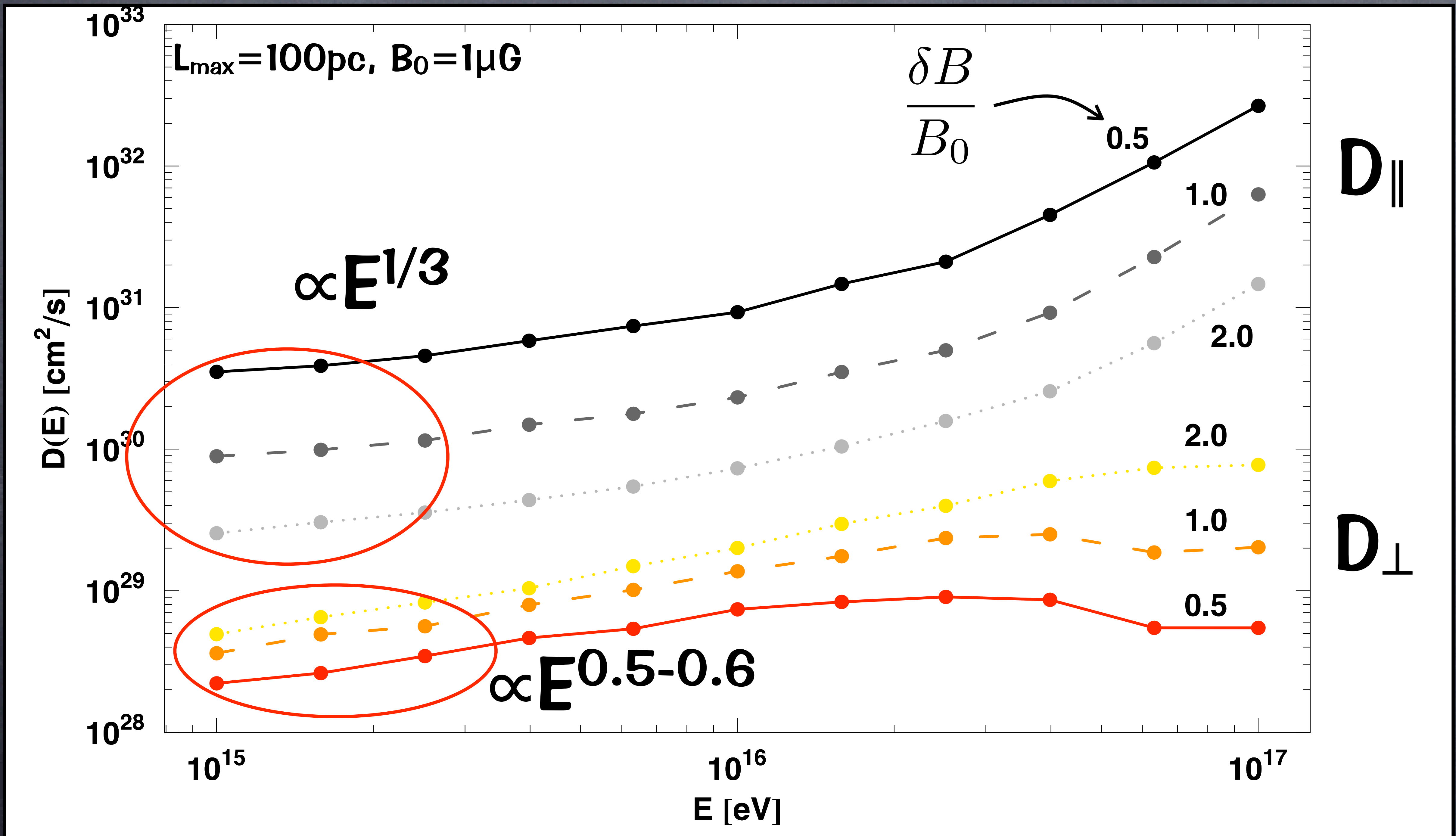




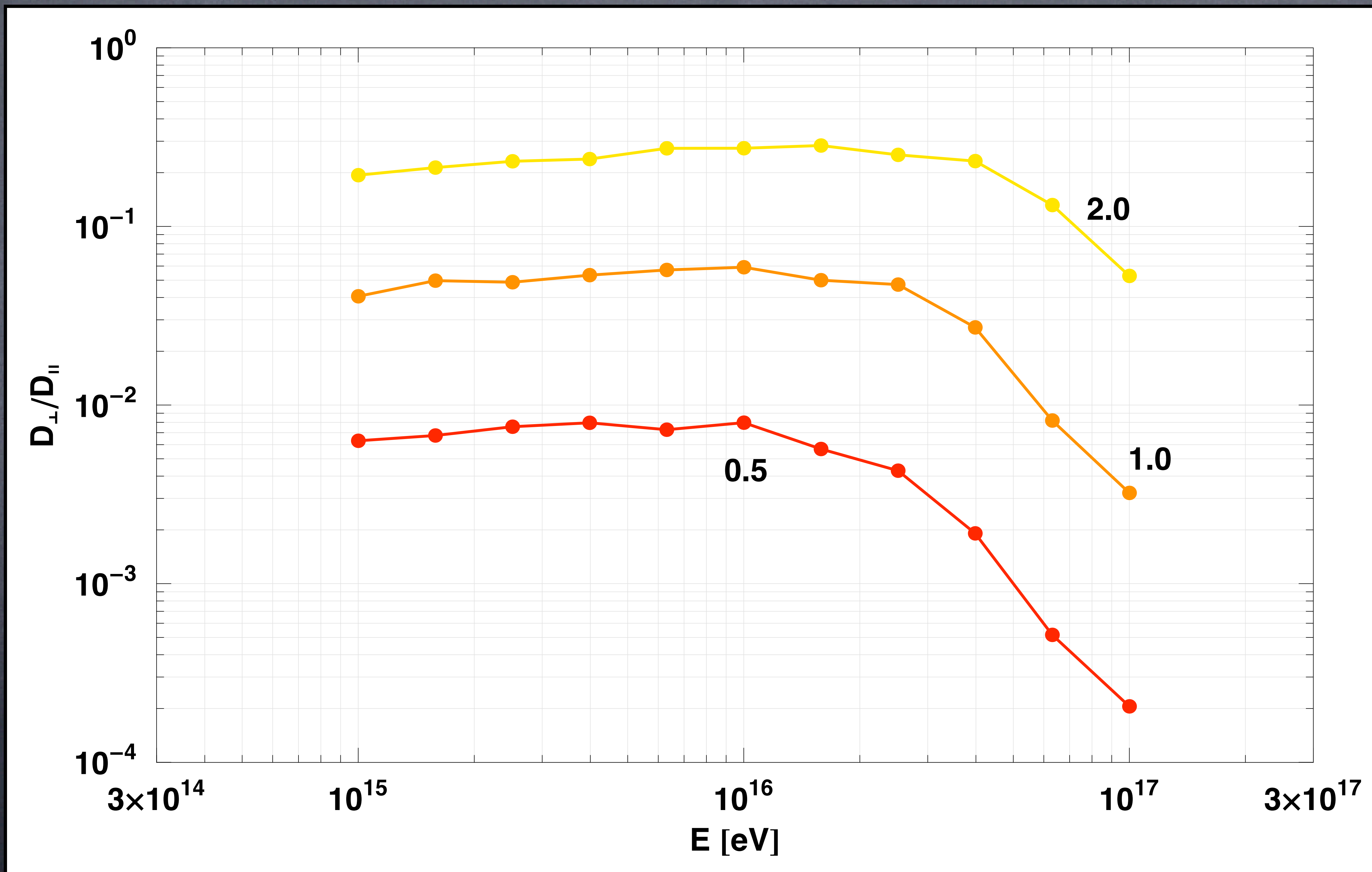
Diffusion Coefficients



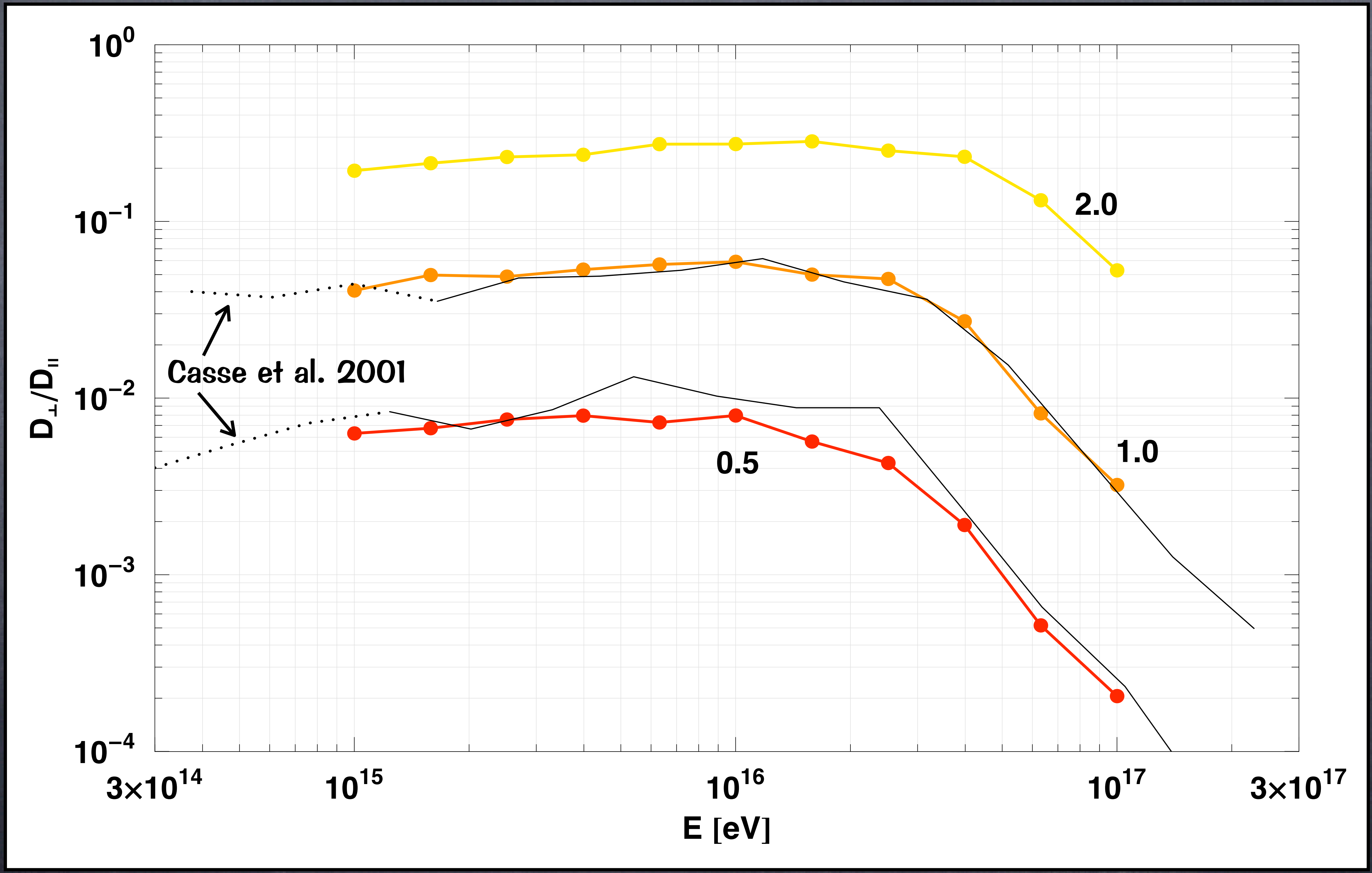
Diffusion Coefficients



D_{perp}/D_{par}


10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$D_\perp/D_{||}$
2.0
1.0
0.5
3×10^{14}
10^{15}
10^{16}
10^{17}
3×10^{17}
E [eV]

Dperp/Dpar



10^0
10^{-1}
10^{-2}
10^{-3}
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$D_\perp/D_\parallel$
2.0
1.0
0.5
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10^{17}
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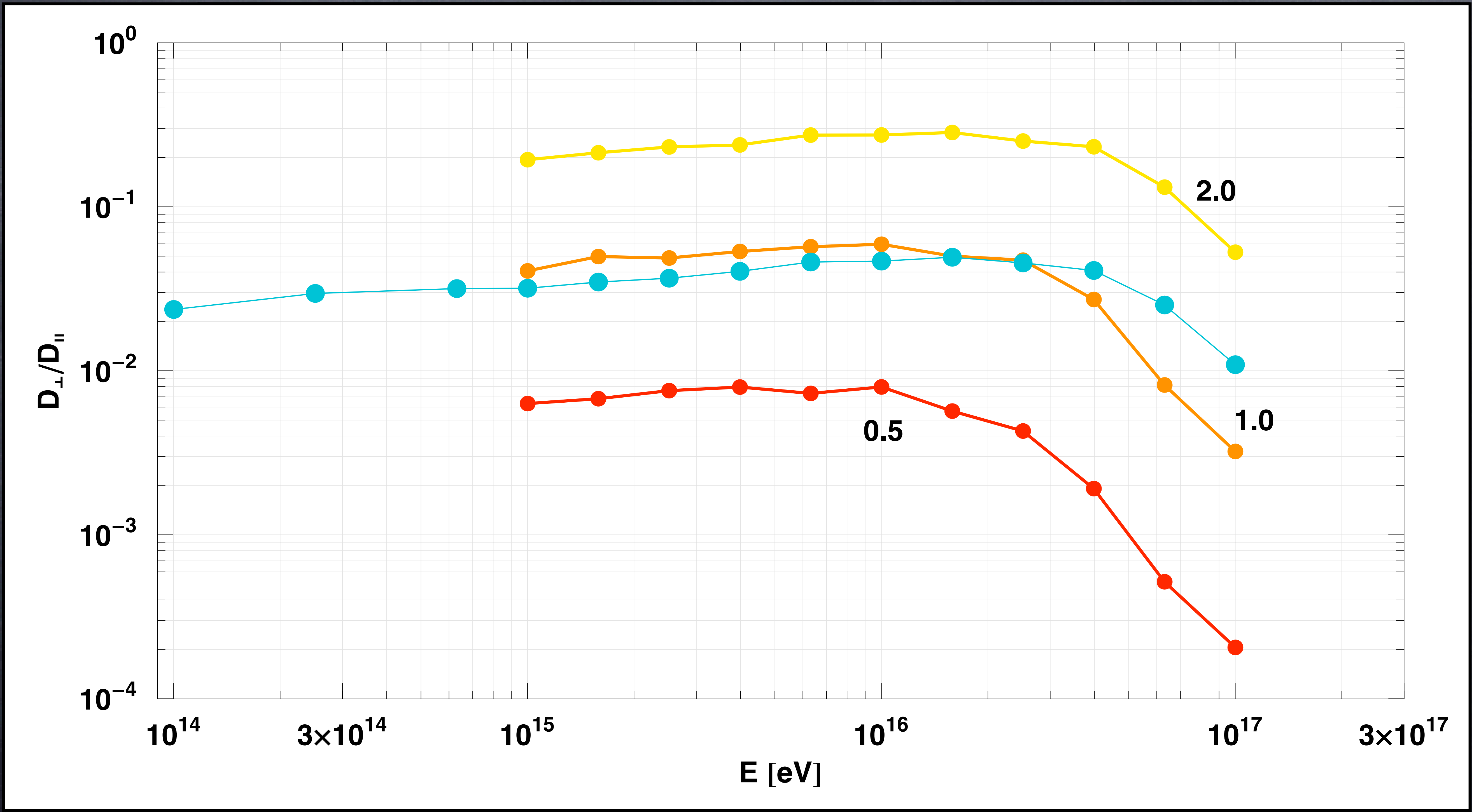
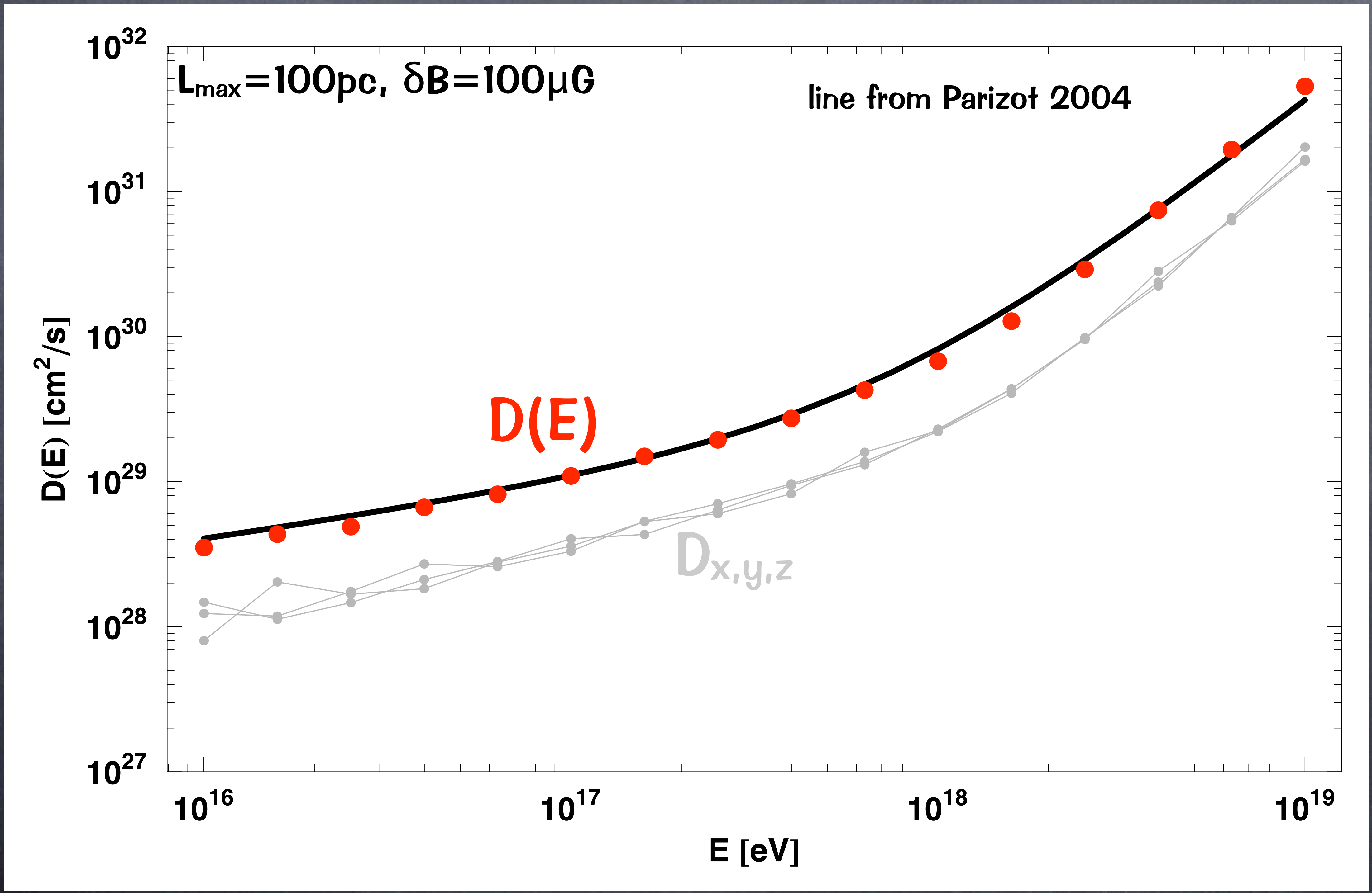
E [eV]

D_{perp}/D_{par}

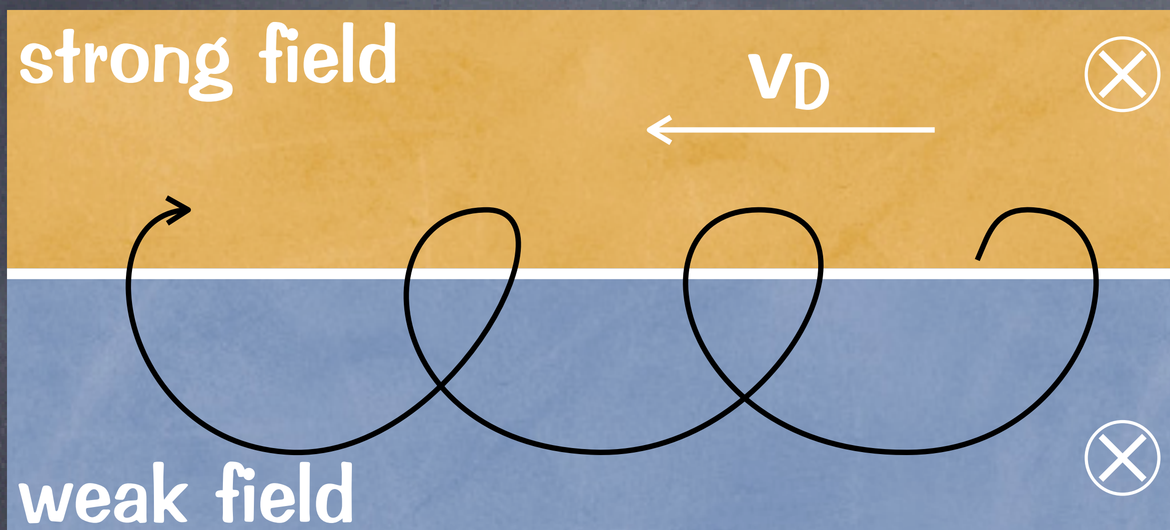


10^0
10^−1
10^−2
10^−3
10^−4
D⊥/D∥
2.0
1.0
0.5
10^14
3×10^14
10^15
10^16
10^17
3×10^17
E [eV]

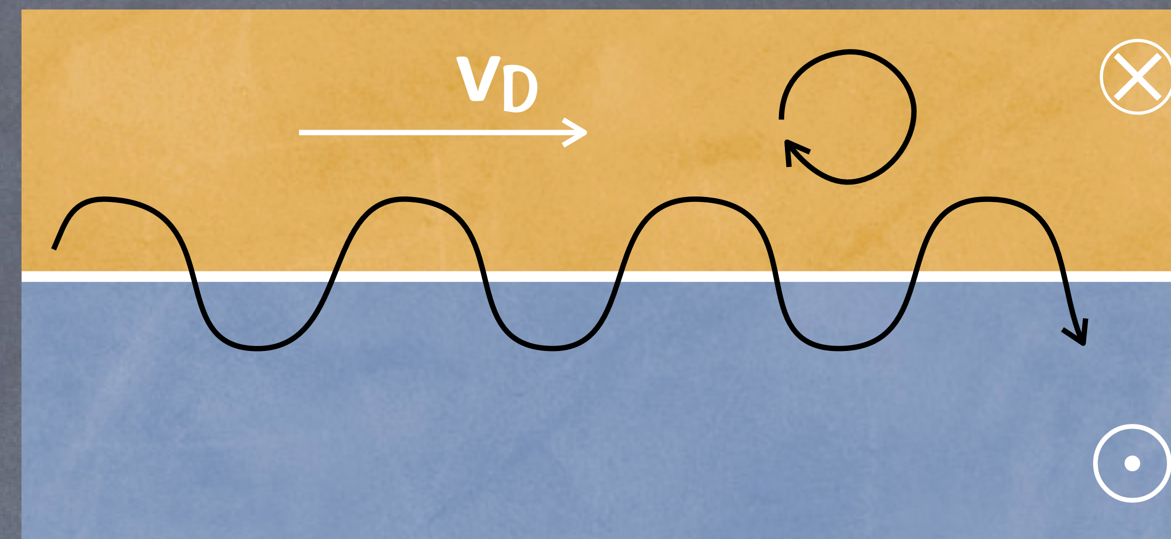
without BG. field



Drifts



gradient drift



curvature drift

turbulence reduces drifts

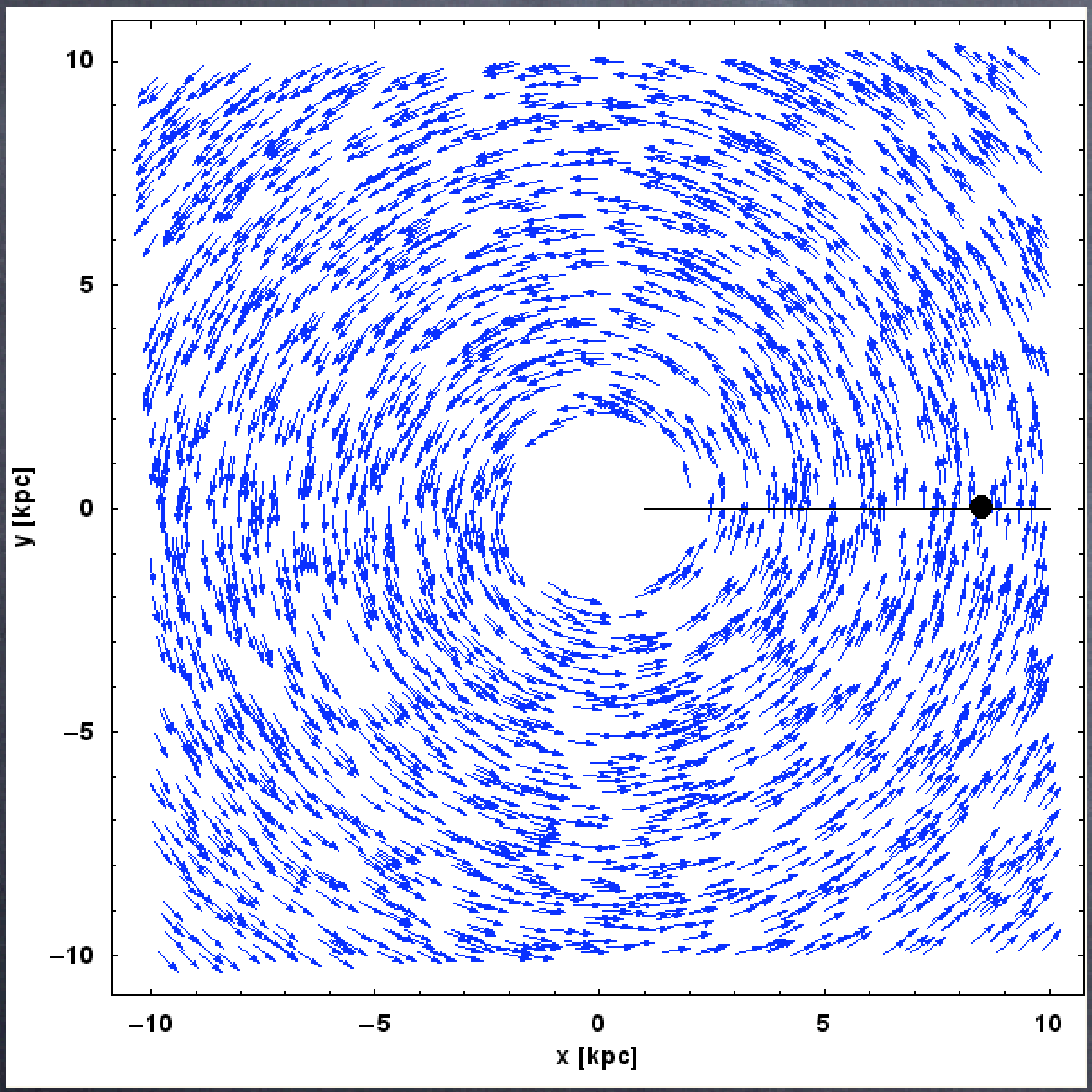
$$\boldsymbol{V}_\perp = c\, r_L \left\{ \frac{1}{2}\sin^2\alpha \frac{\boldsymbol{B_0} \times \boldsymbol{\nabla} B_0}{B_0^2} + \cos^2\alpha \left[\frac{\boldsymbol{B_0} \times \boldsymbol{\nabla} B_0}{B_0^2} + \frac{(\boldsymbol{\nabla} \times \boldsymbol{B_0})_\perp}{B_0} \right] \right\}$$

pitch angle

Rossi 1970

1st order computation: average over a gyration
does not make sense if the field varies on smaller scales

Toy Model: Azimuthal Field

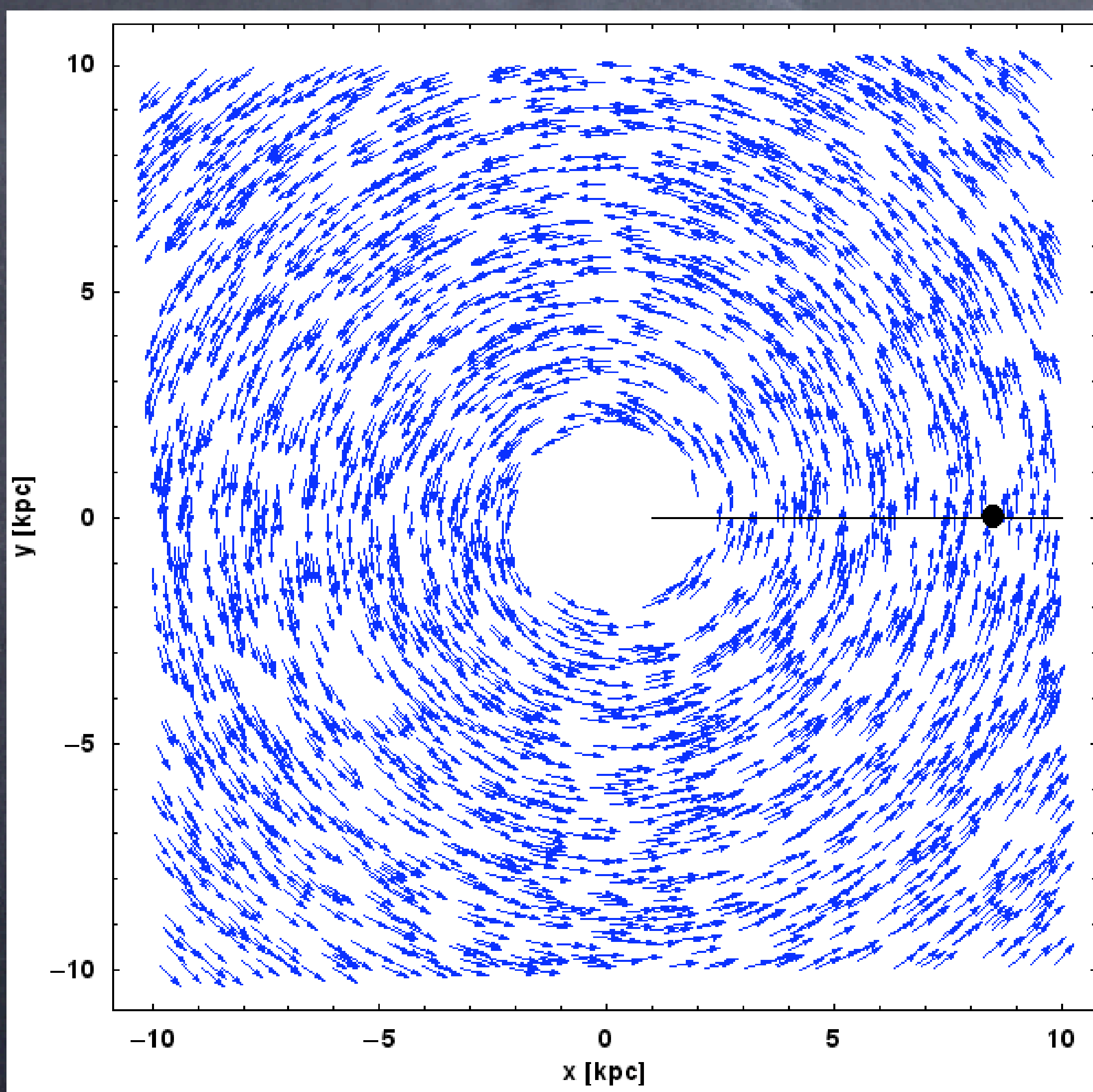


Zirakashvili et al. 1998, Horandel et al. 2007 ...

B=1μG, azimuthal, constant

- field lines are closed: D_{perp}
- D_{par} does not matter
- drifts might be important

Toy Model: Azimuthal Field



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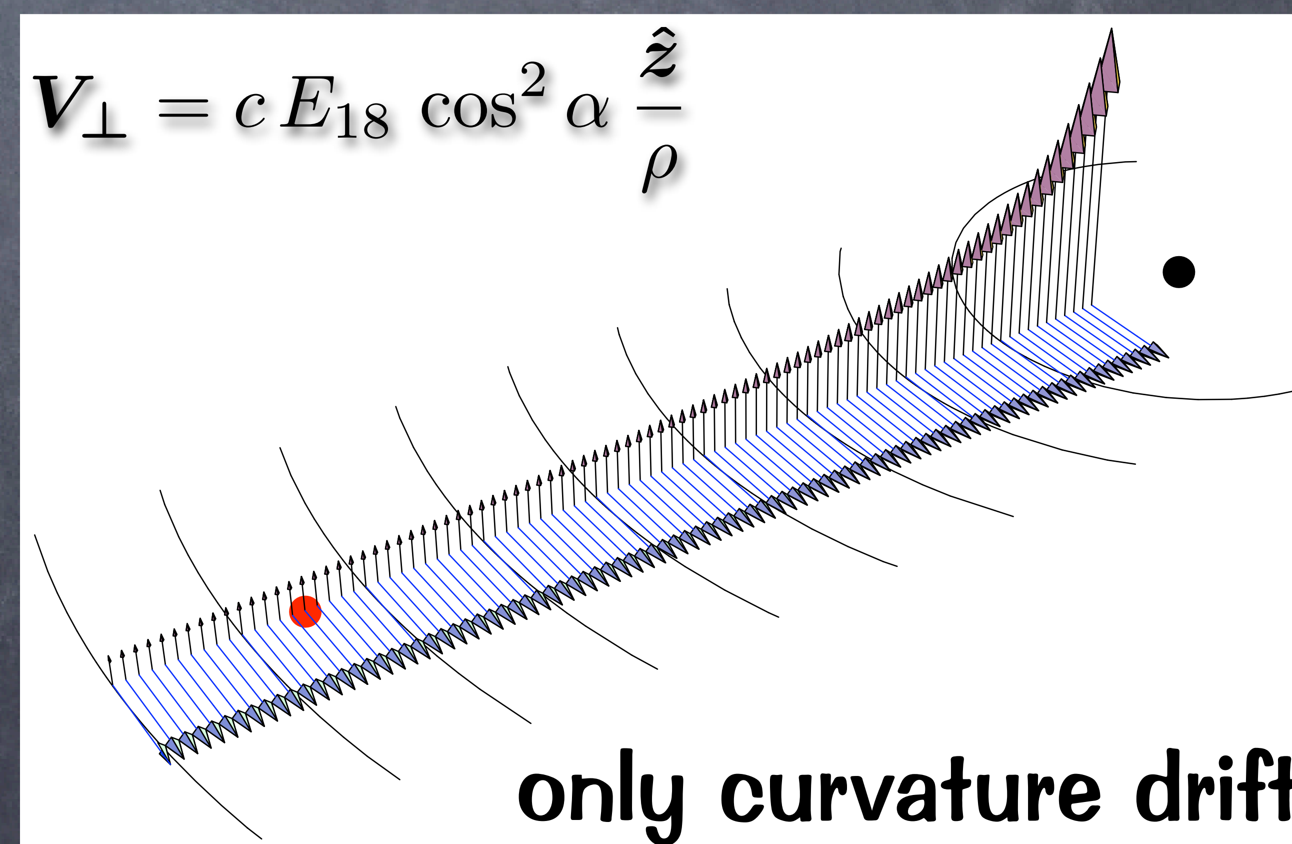
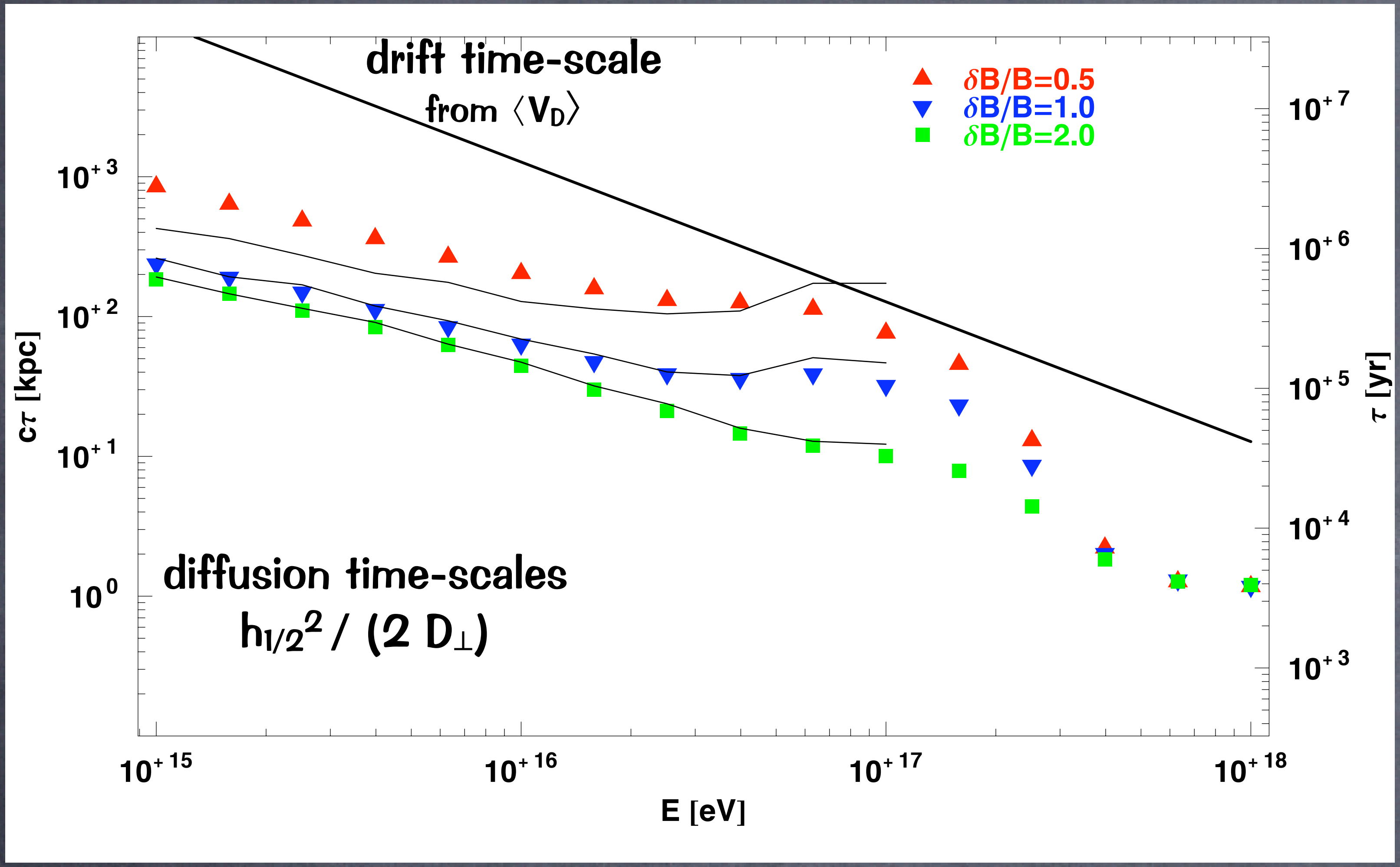
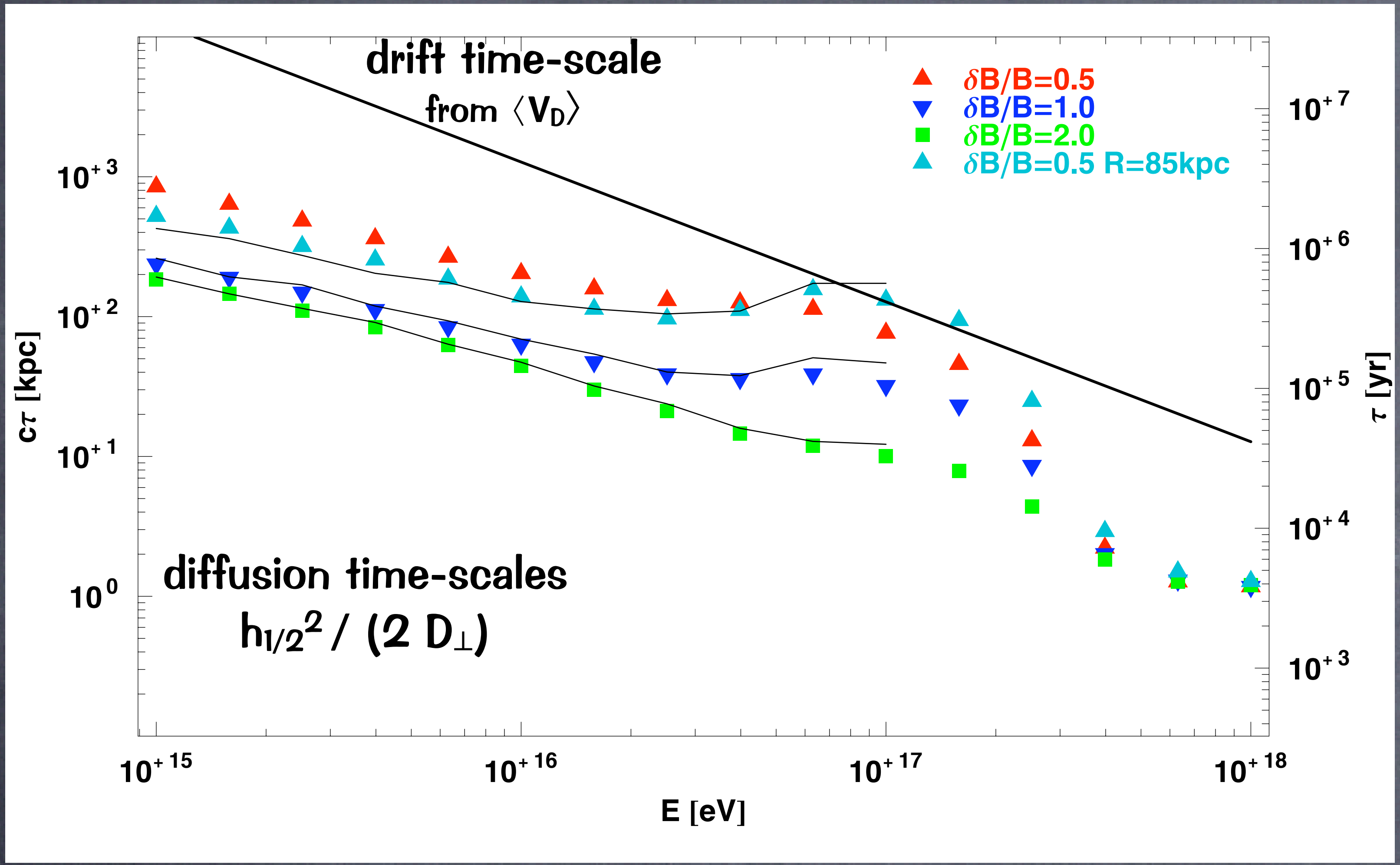


Az. Field: time of escape



time of escape from a cylinder with $h_{1/2}$=0.5kpc

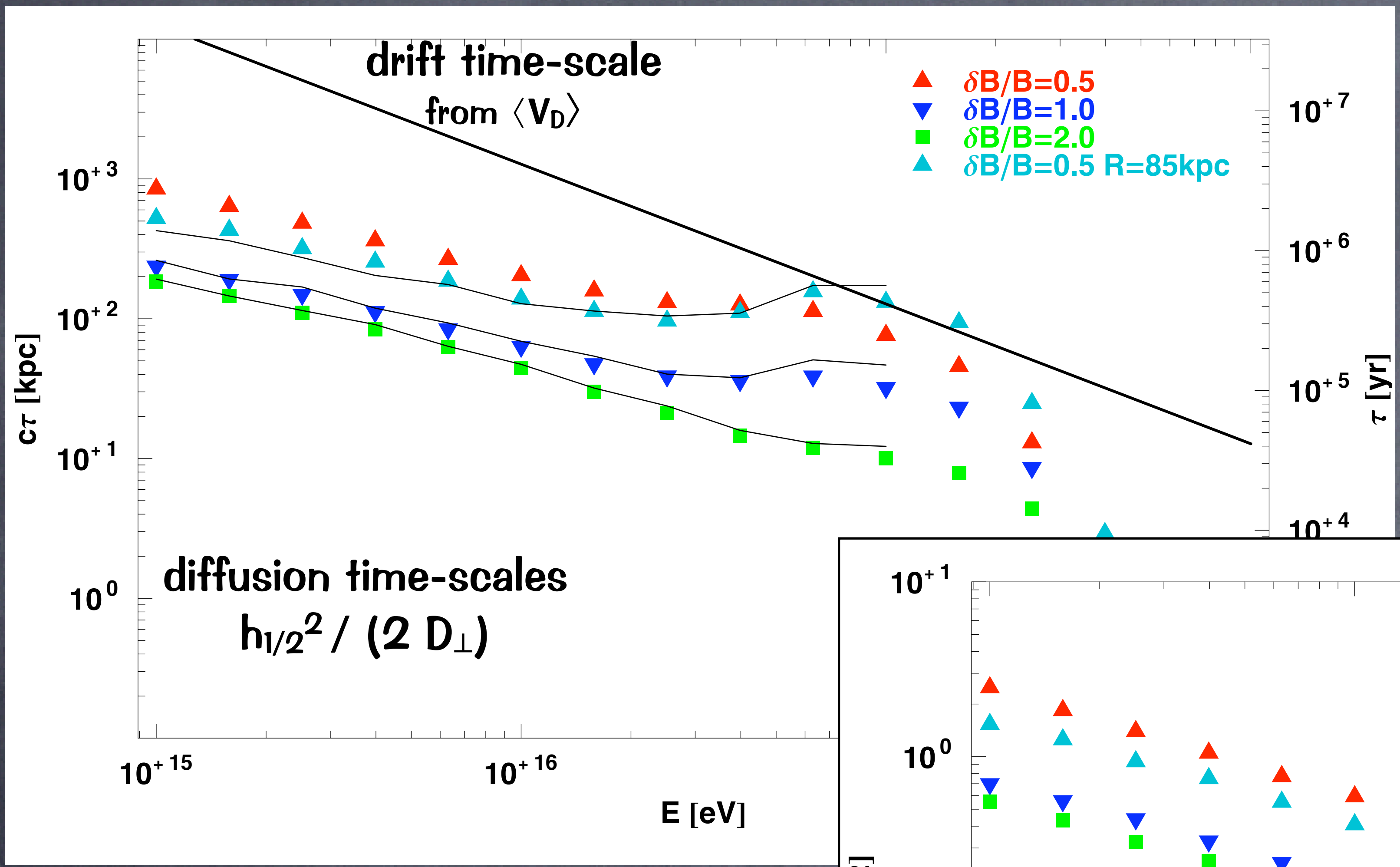
Az. Field: time of escape

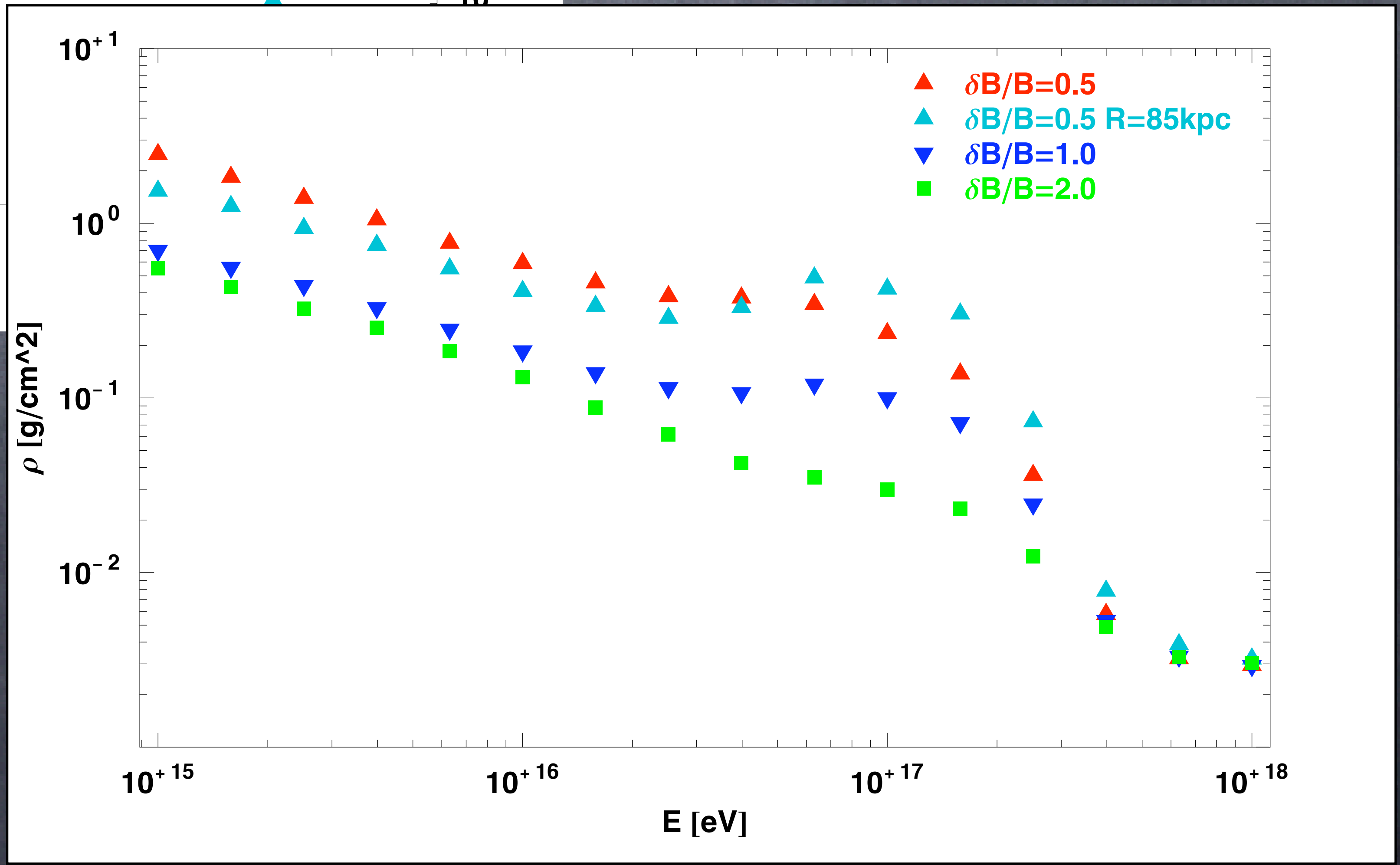


time of escape from a cylinder with $h_{1/2}$=0.5kpc

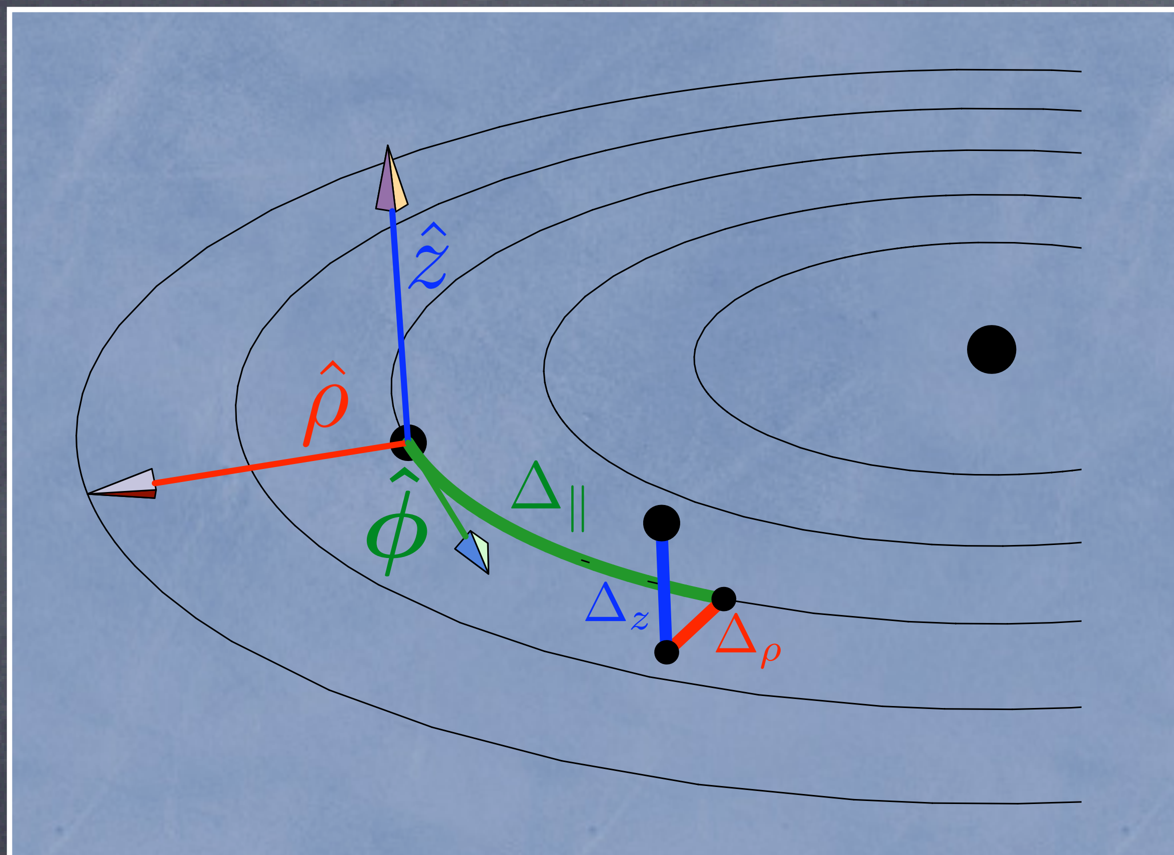
Az. Field: time of escape

time of escape from a cylinder with $h_{1/2}$=0.5kpc





Azimuthal Diffusion



~~$D = \frac{\langle \Delta^2 \rangle}{2\tau}$~~

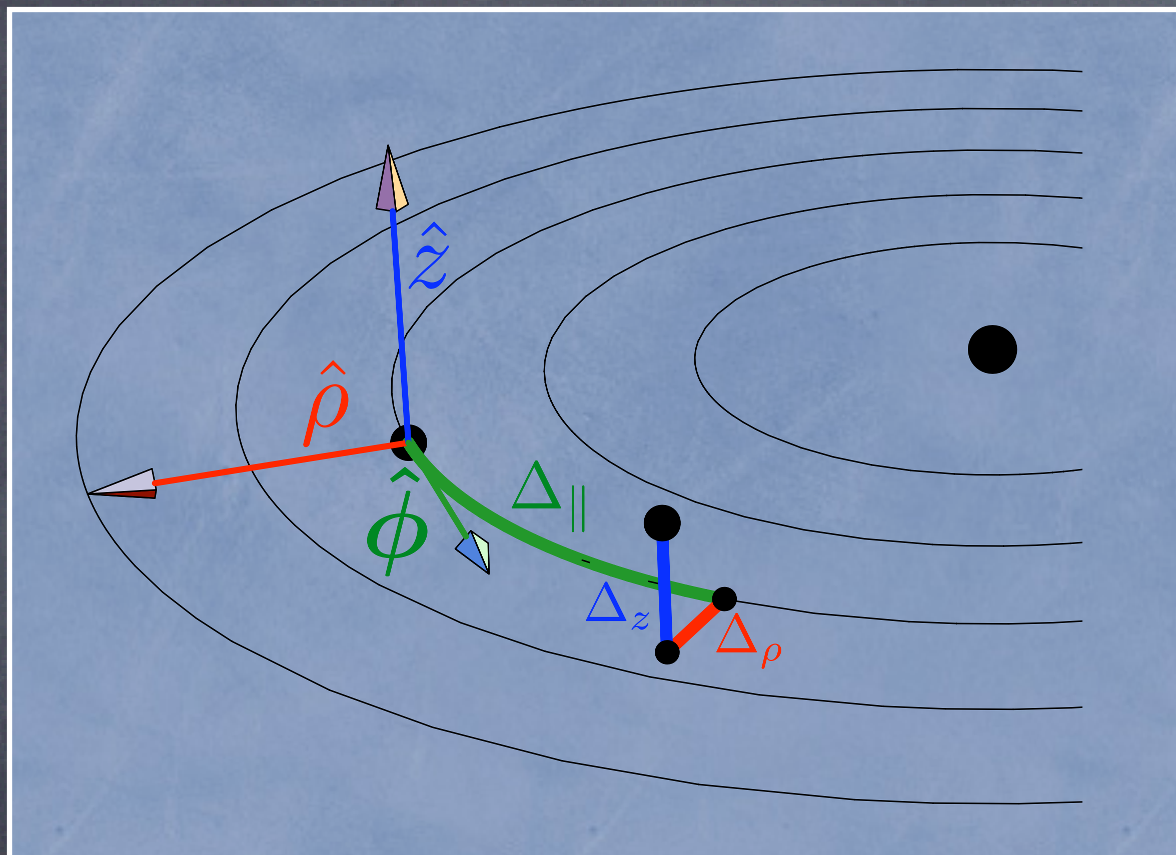
fit with gaussian
at fixed times

$\mu, \ \sigma$

$\mu \rightarrow V_D$

$\sigma \rightarrow D(E)$

Azimuthal Diffusion



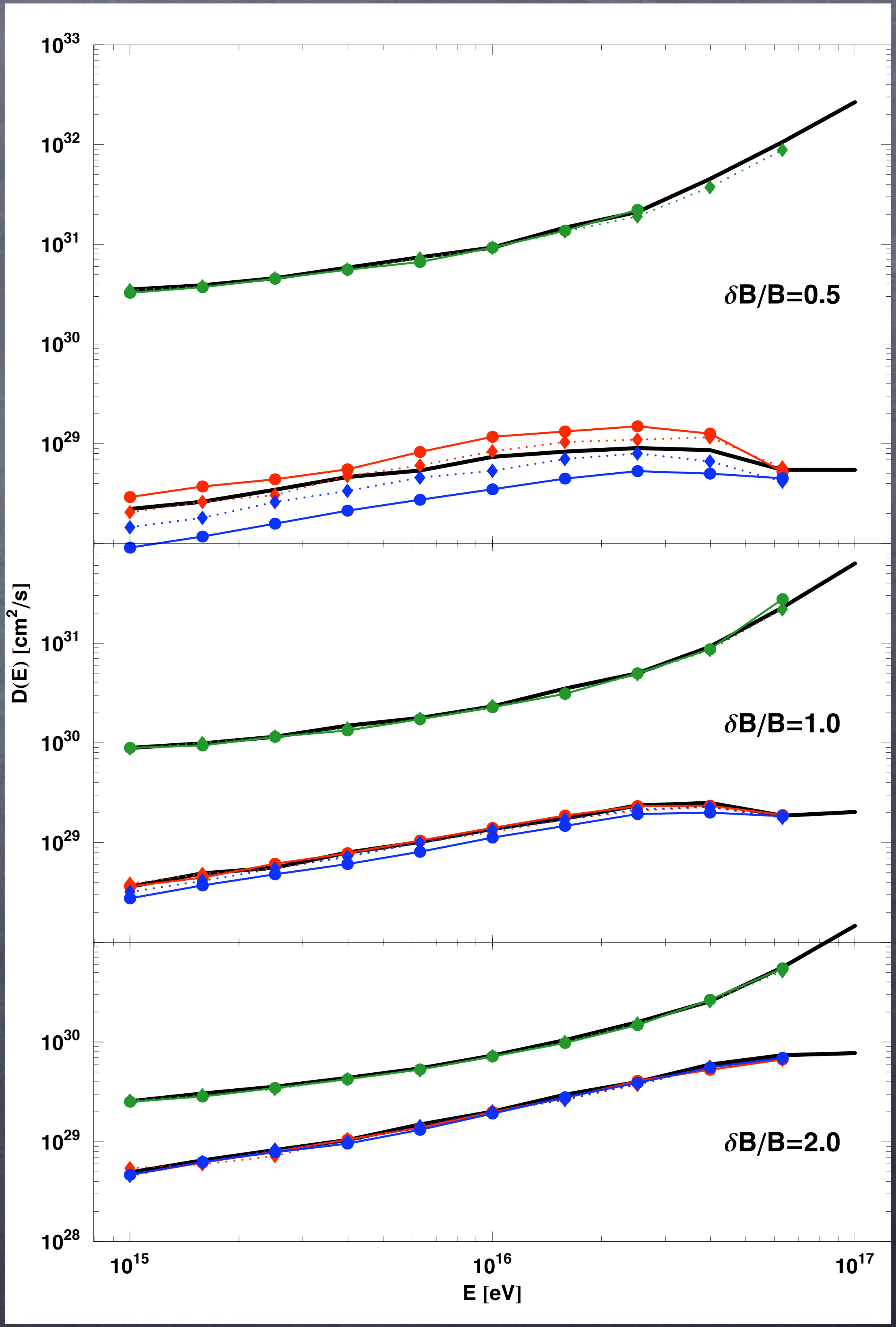
$$D = \frac{\langle \Delta^2 \rangle}{2\tau}$$

fit with gaussian
at fixed times

$\mu, \ \sigma$

V_D ← μ, σ → D(E)

diffusion is modified



"Realistic" Galaxy

$$B(\rho,\theta) = B_0(\rho)\cos\left(\theta - \beta\log\frac{\rho}{\rho_0}\right)$$

β: $1/\tan(p) = -5.67$

ρ_0: 10.55kpc

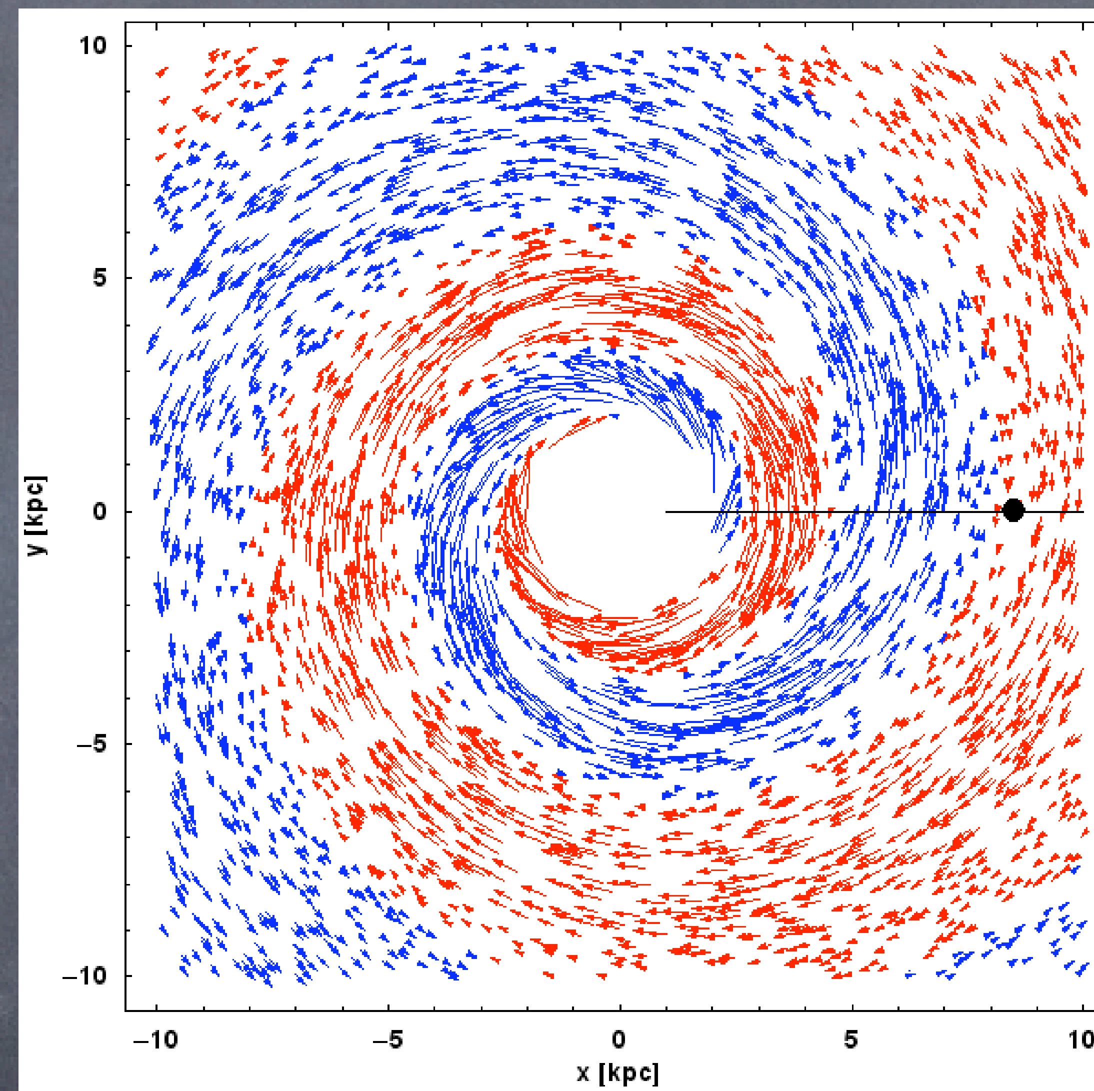
$$B_0(\rho) = 3\,\frac{\rho_\oplus}{\rho}\exp\left(\frac{\rho_\oplus - \rho}{25\text{kpc}}\right)\mu\text{G}$$

$$B(\rho,\theta,z) = \pm B(\rho,\theta)\exp(-|z|/z_0)$$

two z scales:

- z_0=1kpc for $|z|<0.5$kpc
- z_0=4kpc for $|z|>0.5$kpc

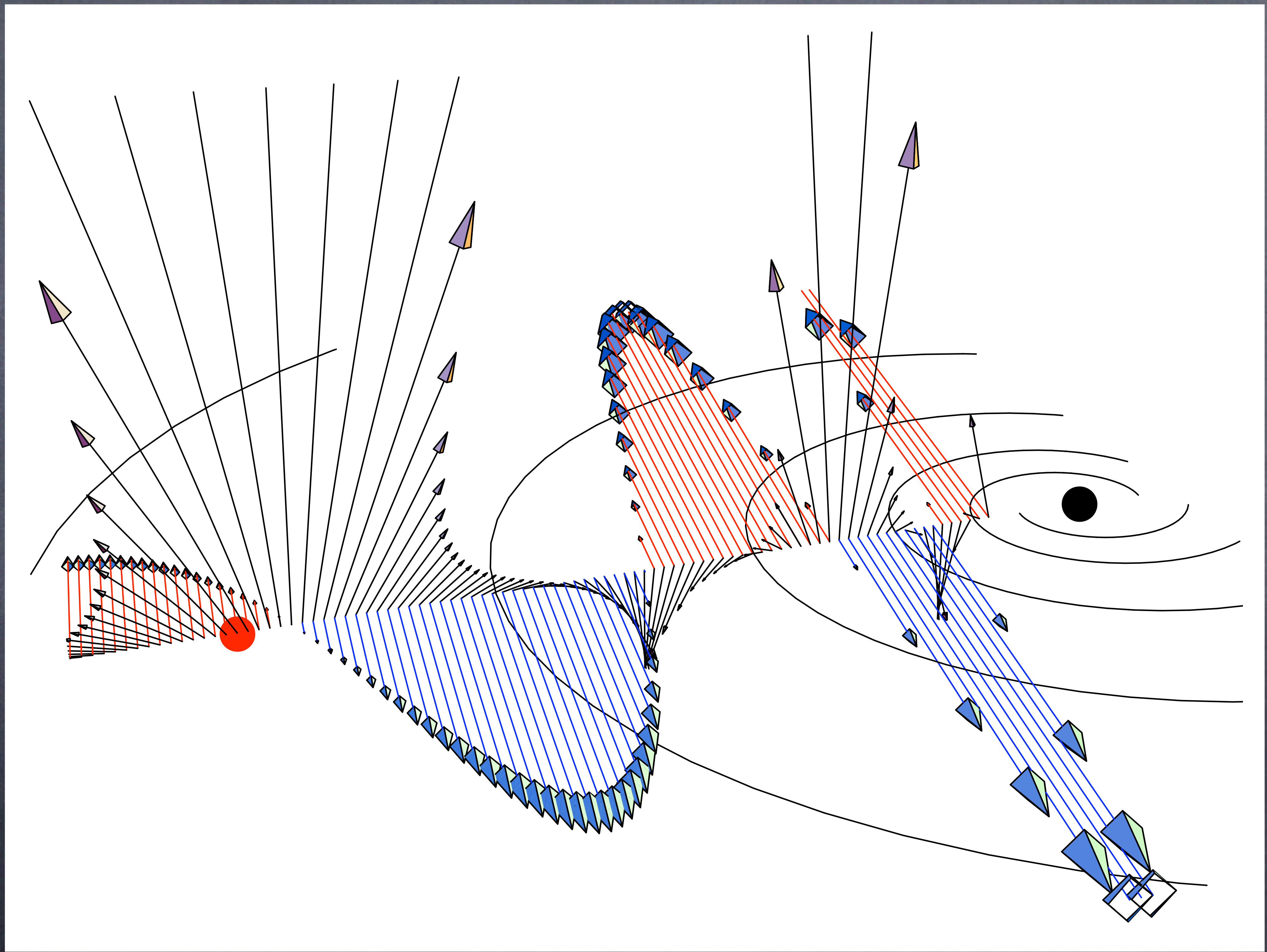
BSS-A



Stanev 1997, Han&Qiao 1994 ...

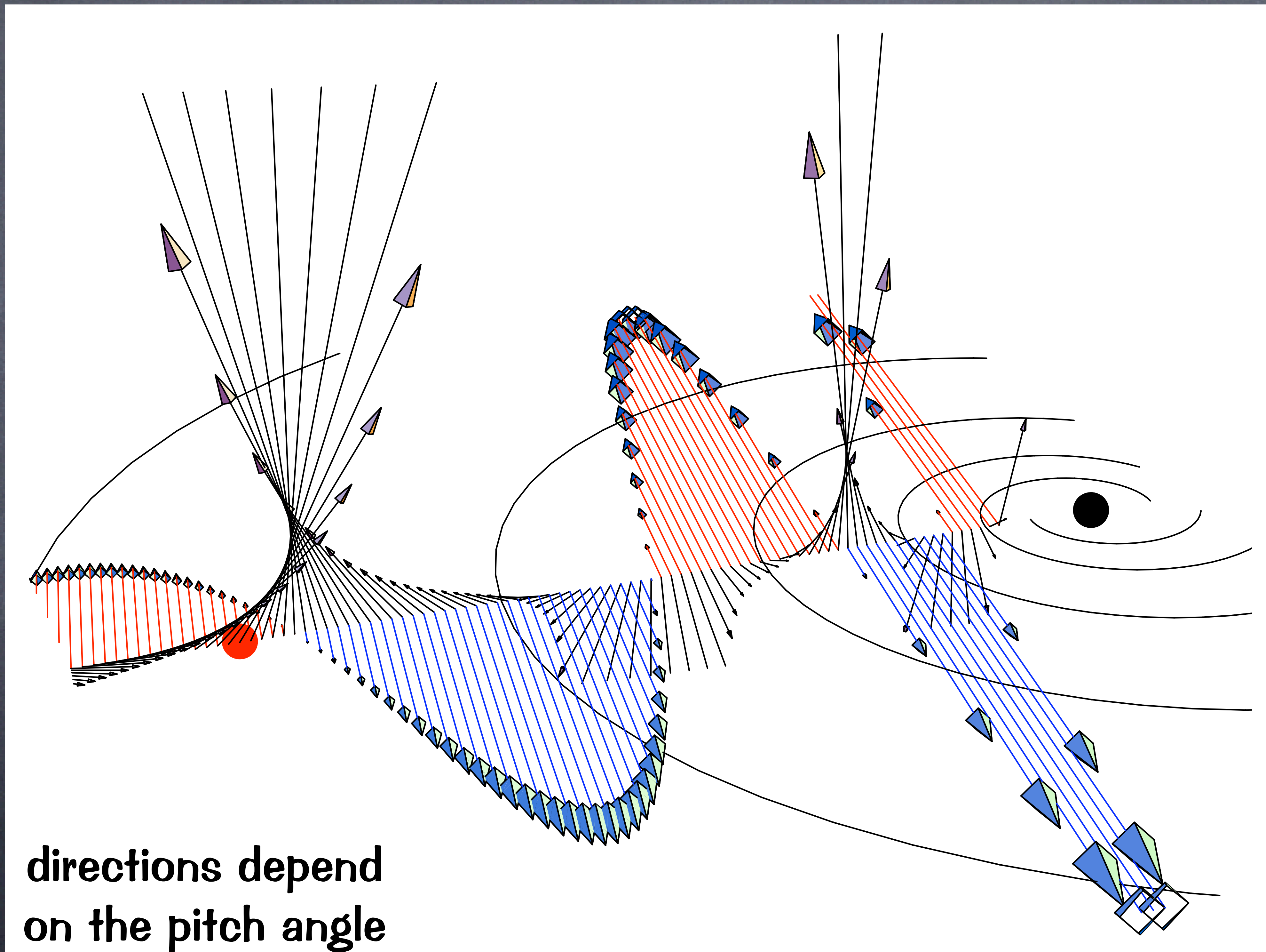
- many scales
- R&Z gradients
- arms gradients

Drifts - BSS Field



just above the plane

Drifts - BSS Field



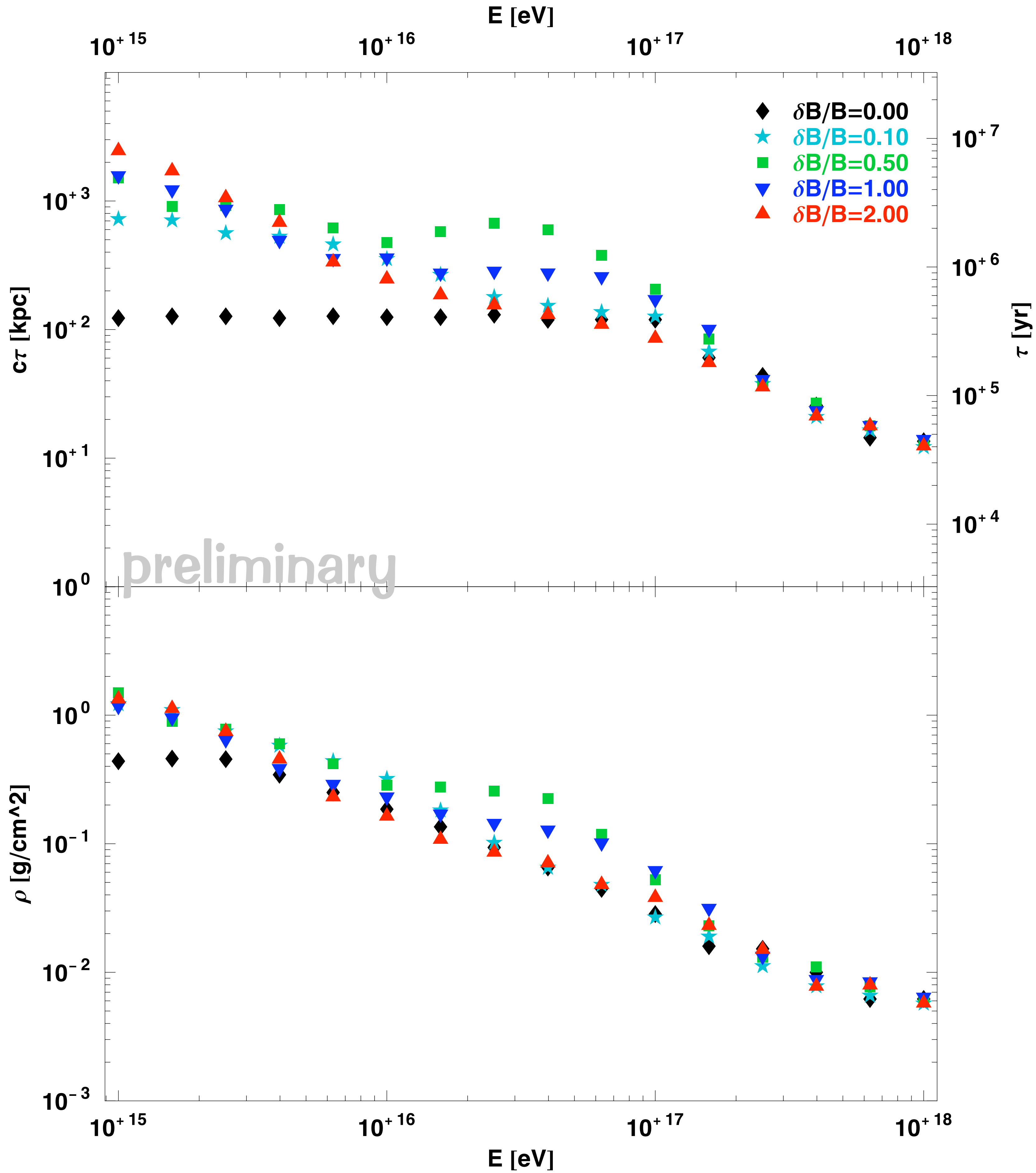
just below the plane

time of escape

injection at Earth,
collection at a cylinder
$h_{1/2}$=4kpc, R=20kpc

δB/B constant

- low energy slopes ~ 0.6-0.8
- absolute values already too large



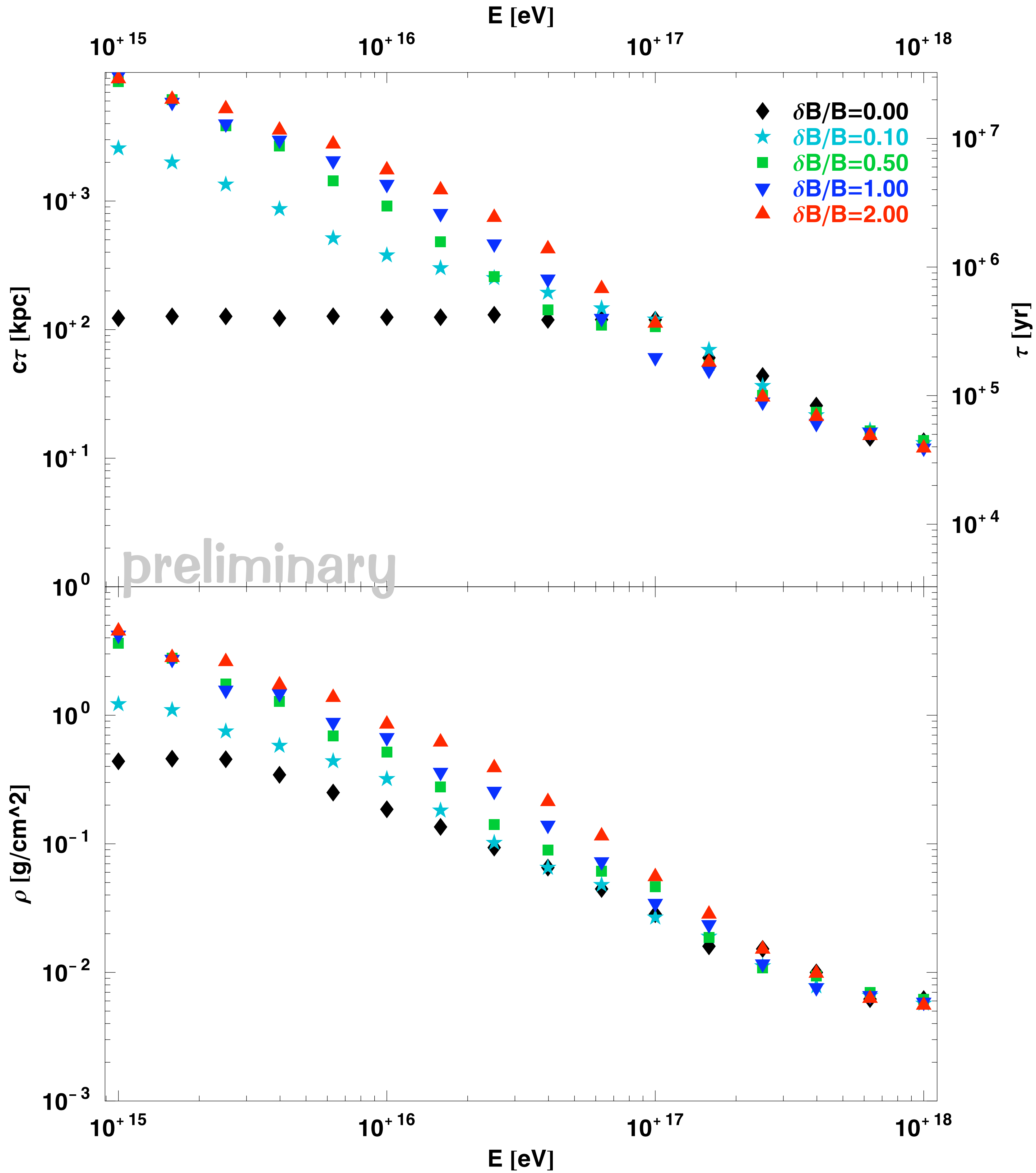
time of escape

injection at Earth,
collection at a cylinder
$h_{1/2}$=4kpc, R=20kpc

δB has no arms

between the arms
δB/B is bigger

- low energy slopes ~ 0.6-0.8
- absolute values already too large



conclusions

- Dperp/Dpar is not constant.
  a kolmogorov spectrum can produce an escape time $E^{-0.5-0.6}$ in some geometries

- the curvature of the background field influences the diffusion process

- for the "realistic" galaxy the flatter slope is not yet seen (down to 10^{15}eV)